EXERCISE – III SUBJECTIVE QUESTIONS

1. Determine the nature of the quadrilateral formed by four lines $3x + 4y - 5 = 0$; $4x - 3y - 5 = 0$; $3x + 4y + 5 = 0$ and $4x - 3y + 5 = 0$. Find the equation of the circle inscribed and circumscribing this quadrilateral.

2. A circle S = 0 is drawn with its centre at (–1, 1) so as to touch the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ externally. Find the intercept made by the circle S = 0 on the coordinate axes.

3. The line $lx + my + n = 0$ intersects the curve $ax^2 + 2hxy + by^2 = 1$ at the point P and Q. The circle on PQ as diameter passes through the origin. Prove that $n^2(a + b) = l^2 + m^2$.

4. One of the diameters of the circle circumscribing the rectangle ABCD is $4y = x + 7$. If A & B are the points (–3, 4) & (5, 4) respectively, then find the area of the rectangle.

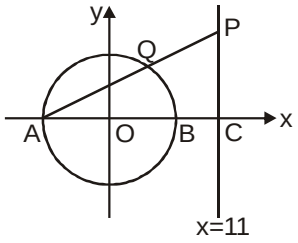
5. Find the equation to the circle which is such that the length of the tangents to it from the points (1, 0), (2, 0) and (3, 2) are 1, $\sqrt{7}$, $\sqrt{2}$ respectively.

6. Let L_1 be a straight line through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 & L_2 are equal, then find the equation(s) which represent L_1.

7. A circle passes through the points (–1, 1), (0, 6) and (5, 5). Find the points on the circle the tangents at which are parallel to the straight line joining origin to the centre.

8. Find the equations of straight lines which pass through the intersection of the lines $x - 2y - 5 = 0$, $7x + y = 50$ & divide the circumference of the circle $x^2 + y^2 = 100$ into two arcs whose lengths are in the ratio 2 : 1.

9. In the given figure, the circle $x^2 + y^2 = 25$ intersects the x-axis at the point A and B. The line x = 11 intersects the x-axis at the point C. Point P moves along the line x = 11 above the x-axis and AP intersects the circle at Q. Find



(i) The coordinates of the point P if the triangle AQB has the maximum area.

(ii) The coordinates of the point P if Q is the middle point of AP.

(iii) The coordinates of P if the area of the triangle AQB is $(1/4)^{th}$ of the area of the triangle APC.

10. A circle is drawn with its centre on the line x+y=2 to touch the line $4x - 3y + 4 = 0$ and pass through the point (0, 1). Find its equation.

11. A point moving around circle $(x + 4)^2 + (y + 2)^2 = 25$ with centre C broke away from it either at the point A or point B on the circle and moved along a tangent to the circle passing through the point D(3, –3).

Find the following.

(i) Equation of the tangents at A and B.

(ii) Coordinates of the points A and B.

(iii) Angle ADB and the maximum and minimum distances of the point D from the circle.

(iv) Area of quadrilateral ADBC and the ΔDAB.

(v) Equation of the circle circumscribing the ΔDAB and also the intercepts made by this circle on the coordinate axes.

12. Find the locus of the mid point of the chord of a circle $x^2 + y^2 = 4$ such that the segment intercepted by the chord on the curve $x^2 - 2x - 2y = 0$ subtends a right angle at the origin.

13. Find the equation of a line with gradient 1 such that the two circles $x^2+y^2=4$ and $x^2+y^2-10x-14y+65=0$ intercept equal length on it.

14. Find the locus of the middle points of portions of the tangents to the circle $x^2 + y^2 = a^2$ terminated by the coordinates axes.

15. Tangents are drawn to the concentric circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ at right angle to one another. Show that the locus of their point of intersection is a 3rd concentric circle. Find its radius.

16. Find the equation of the circle passing through the three points (4, 7), (5, 6) and (1, 8). Also find the coordinates of the point of intersection of the tangents to the circle at the points where it is cut by the straight line $5x + y + 17 = 0$.

17. Consider a circle S with centre at the origin and radius 4. Four circles A, B, C and D each with radius unity and centres (–3, 0), (–1, 0), (1, 0) and (3, 0) respectively are drawn. A chord PQ of the circle S touches the circle B and passes through the centre of the circle C. If the length of this chord can be expressed as $\sqrt{x}$, find x.

18. Obtain the equations of the straight lines passing through the point A(2, 0) & making 45º angle with the tangent at A to the circle $(x + 2)^2 + (y - 3)^2 = 25$. Find the equations of the circles each of radius 3 whose centres are on these straight lines at a distance of $5\sqrt{2}$ from A.

19. The lines $2x - 3y + 1 = 0$ is tangent to a circle S=0 at (1, 1). If the radius of the circle is $\sqrt{13}$. Find the equation of the circle S.

20. Find the equation of the circle which passes through the point (1, 1) & which touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point (2, 3) on it.

21. Let **K** denotes the square of the diameter of the circle whose diameter is the common chord of the two circles $x^2 + y^2 + 2x + 3y + 1 = 0$ and $x^2 + y^2 + 4x + 3y + 2 = 0$ and **W** denotes the sum of the abscissa and ordinate of a point P where all variable chords of the curve $y^2 = 8x$ subtending right angles at the origin, are concurrent.
and **H** denotes the square of the length of the tangent from the point (3, 0) on the circle $2x^2 + 2y^2 + 5y - 16 = 0$. Find the value of KWH.

22. Show that the equation of a straight line meeting the circle $x^2 + y^2 = a^2$ in two points at equal distance 'd' from a point (x_1, y_1) on its circumference is $xx_1 + yy_1 - a^2 + (d^2/2) = 0$.

23. The radical axis of the circles $x^2+y^2+2gx+2fy+c=0$ and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x - 2y + 1 = 0$. Show that either $g = 3/4$ or $f = 2$.

24. Find the equation of the circle through the points of intersection of circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$ & cutting the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.

25. The centre of the circle $S = 0$ lie on the line $2x - 2y + 9 = 0$ & $S = 0$ cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle $S = 0$ passes through two fixed points & find their coordinates.

26. (a) Find the equation of a circle passing through the origin if the line pair, $xy - 3x + 2y - 6 = 0$ is orthogonal to it. If this circle is orthogonal to the circle $x^2 + y^2 - kx + 2ky - 8 = 0$ then find the value of k.

(b) Find the equation of the circle which cuts the circle $x^2 + y^2 - 14x - 8y + 64 = 0$ and the coordinates axes orthogonally.

27. Find the equation of the circle whose radius is 3 and which touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ internally at the point (–1, –1).

28. Show that the locus of the centres of a circle which cuts two given circles orthogonally is a straight line & hence deduce the locus of the centres of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ & $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally. Interpret the locus.